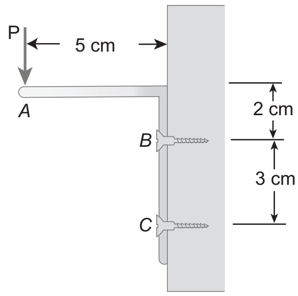
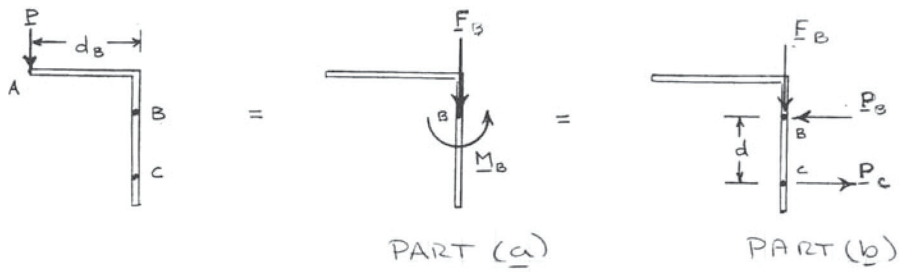


PROBLEM 3.82

A 30-N vertical force P is applied at A to the bracket shown, which is held by screws at B and C. (a) Replace **P** with an equivalent force couple system at B. (b) Find the two horizontal forces at B and C that are equivalent to the couple obtained in part a.

SOLUTION



(a) $\Sigma\mathbf{F}$: $\mathbf{F}_B = 30\text{ N}$

or $\mathbf{F}_B = 30\text{ N}\downarrow$ ◀

$\Sigma\mathbf{M}$: $M_B = P\,d_B$

$= (30\text{ N})\,(0.05\text{ m})$

$= 1.5\text{ N}\cdot\text{m}$

or $\mathbf{M}_B = 1.5\text{ N}\cdot\text{m}$ ◀

(b) $\Sigma\mathbf{M}_B$: $M_B = F_C\,d$

$1.5\text{ N}\cdot\text{m} = F_C(0.03\text{ m})$

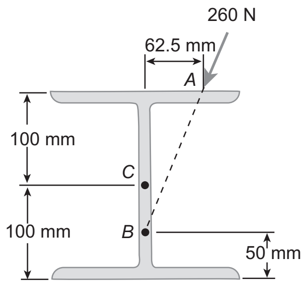
$F_C = 50\text{ N}$

or $\mathbf{F}_C = 50\text{ N}\longrightarrow$ ◀

$\Sigma\mathbf{F}$: $0 = -F_B + F_C$

$F_B = F_C = 50\text{ N}$

or $\mathbf{F}_B = 50\text{ N}\longleftarrow$ ◀



PROBLEM 3.81

A 260-N force is applied at A to the rolled-steel section shown. Replace that force with an equivalent force-couple system at the center C of the section.

SOLUTION

$$AB = \sqrt{(0.0625\text{ m})^2 + (0.15\text{ m})^2} = 0.1625\text{ m}$$

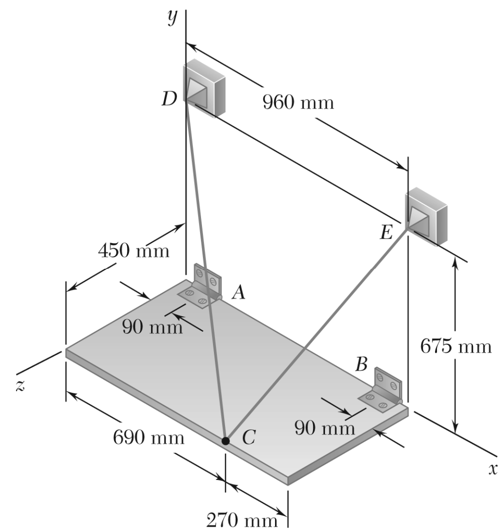
$$\sin\alpha = \frac{0.0625\text{ m}}{0.1625\text{ m}} = \frac{5}{13}$$

$$\cos\alpha = \frac{0.15\text{ m}}{0.1625\text{ m}} = \frac{12}{13} \qquad \alpha = 22.6°$$

$$\begin{aligned}\mathbf{F} &= -F\sin\alpha\mathbf{i} - F\sin\alpha\mathbf{j}\\ &= -(260\text{ N})\frac{5}{13}\mathbf{i} - (260\text{ N})\frac{12}{13}\mathbf{j}\\ &= -(100.0\text{ N})\mathbf{i} - (240\text{ N})\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{M}_C &= \mathbf{r}_{A/C} \times \mathbf{F}\\ &= (0.0625\mathbf{i} + 0.1\mathbf{j}) \times (-100.0\mathbf{i} - 240\mathbf{j})\\ &= 10\mathbf{k} - 15\mathbf{k}\\ &= -(5\text{ N}\cdot\text{m})\mathbf{k}\end{aligned}$$

$\mathbf{F} = 260\text{ N} \measuredangle 67.4°; \quad \mathbf{M}_C = 5\text{ N}\cdot\text{m}$ ◀

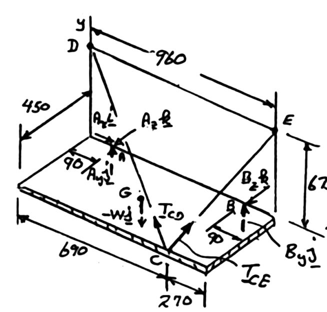


PROBLEM 4.113

A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

Dimensions in mm



$$\mathbf{r}_{B/A}(960-180)\mathbf{i} = 780\mathbf{i}$$

$$\mathbf{r}_{G/A} = \left(\frac{960}{2}-90\right)\mathbf{i} + \frac{450}{2}\mathbf{k}$$

$$= 390\mathbf{i} + 225\mathbf{k}$$

$$\mathbf{r}_{C/A} = 600\mathbf{i} + 450\mathbf{k}$$

T = Tension in cable *DCE*

$$\overrightarrow{CD} = -690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CD = 1065 \text{ mm}$$

$$\overrightarrow{CE} = 270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CE = 855 \text{ mm}$$

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100\text{ kg})(9.81\text{ m/s}^2)\mathbf{j} = -(981\text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A}\times\mathbf{T}_{CD} + \mathbf{r}_{C/A}\times\mathbf{T}_{CE} + \mathbf{r}_{G/A}\times(-W\mathbf{j}) + \mathbf{r}_{B/A}\times\mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ -690 & 675 & -450 \end{vmatrix}\frac{T}{1065} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix}\frac{T}{855}$$

$$+\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

PROBLEM 4.113 (Continued)

Coefficient of **i**: $-(450)(675)\dfrac{T}{1065} - (450)(675)\dfrac{T}{855} + 220.73\times 10^3 = 0$

$T = 344.64 \text{ N}$ $\qquad$ $T = 345 \text{ N}$ ◀

Coefficient of **j**: $(-690\times 450 + 600\times 450)\dfrac{344.64}{1065} + (270\times 450 + 600\times 450)\dfrac{344.64}{855} - 780B_z = 0$

$B_z = 185.516 \text{ N}$

Coefficient of **k**: $(600)(675)\dfrac{344.64}{1065} + (600)(675)\dfrac{344.64}{855} - 382.59\times 10^3 + 780B_y = 0 \quad B_y = 113.178 \text{ N}$

$\mathbf{B} = (113.2 \text{ N})\mathbf{j} + (185.5 \text{ N})\mathbf{k}$ ◀

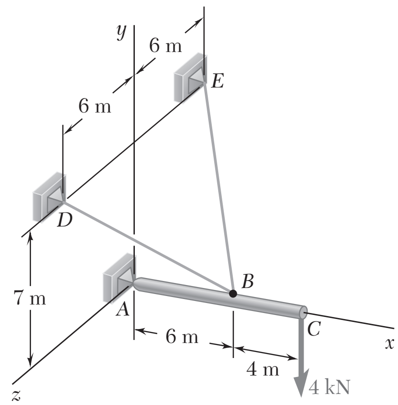
$\Sigma\mathbf{F} = 0\text{:} \quad \mathbf{A} + \mathbf{B} + \mathbf{T}_{CD} + \mathbf{T}_{CE} + \mathbf{W} = 0$

Coefficient of **i**: $A_x - \dfrac{690}{1065}(344.64) + \dfrac{270}{855}(344.64) = 0$ $\qquad$ $A_x = 114.5 \text{ N}$

Coefficient of **j**: $A_y + 113.178 + \dfrac{675}{1065}(344.64) + \dfrac{675}{855}(344.64) - 981 = 0$ $\qquad$ $A_y = 377 \text{ N}$

Coefficient of **k**: $A_z + 185.516 - \dfrac{450}{1065}(344.64) - \dfrac{450}{855}(344.64) = 0$ $\qquad$ $A_z = 141.5 \text{ N}$

$\mathbf{A} = (114.5 \text{ N})\mathbf{i} + (377 \text{ N})\mathbf{j} + (144.5 \text{ N})\mathbf{k}$ ◀



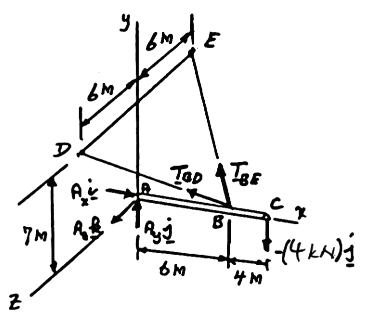
PROBLEM 4.107

A 10-m boom is acted upon by the 4-kN force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at A.

SOLUTION

We have five unknowns and six Eqs. of equilibrium but equilibrium is maintained $(\Sigma M_x = 0)$.

Free-Body Diagram:



$$\overrightarrow{BD} = (-6\text{ m})\mathbf{i} + (7\text{ m})\mathbf{j} + (6\text{ m})\mathbf{k} \qquad BD = 11\text{ m}$$

$$\overrightarrow{BE} = (-6\text{ m})\mathbf{i} + (7\text{ m})\mathbf{j} - (6\text{ m})\mathbf{k} \qquad BE = 11\text{ m}$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k})$$

$$\Sigma M_A = 0\text{:}\ \mathbf{r}_{B/A} \times T_{BD} + \mathbf{r}_{B/A} \times T_{BE} + \mathbf{r}_{C/A} \times (-4\mathbf{j}) = 0$$

$$6\mathbf{i} \times \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}) + 6\mathbf{i} \times \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (-4\mathbf{j}) = 0$$

$$\frac{42}{11}T_{BD}\mathbf{k} - \frac{36}{11}T_{BD}\mathbf{j} + \frac{42}{11}T_{BE}\mathbf{k} + \frac{36}{11}T_{BE}\mathbf{j} - 40\mathbf{k}$$

Equate coefficients of unit vectors to zero.

$$\mathbf{j}\text{:}\quad -\frac{36}{11}T_{BD} + \frac{36}{11}T_{BE} = 0 \quad T_{BE} = T_{BD}$$

$$\mathbf{k}\text{:}\quad \frac{42}{11}T_{BD} + \frac{42}{11}T_{BE} - 40 = 0$$

$$2\left(\frac{42}{11}T_{BD}\right) = 40$$

$T_{BD} = 5.24$ kN ◀

$$T_{BD} = 5.38\text{ m}$$

$T_{BE} = 5.24$ kN ◀

PROBLEM 4.107 (Continued)

$$\Sigma F_x = 0: \quad A_x - \frac{6}{11}(5.24\text{ kN}) - \frac{6}{11}(5.24\text{ kN}) = 0$$

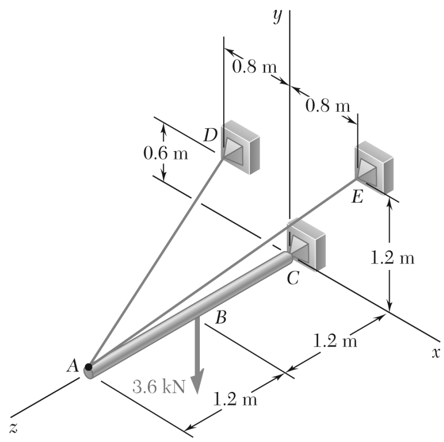
$$A_x = 5.72\text{ kN}$$

$$\Sigma F_y = 0: \quad A_y + \frac{7}{11}(5.24\text{ kN}) + \frac{7}{11}(5.24\text{ kN}) - 4\text{ kN} = 0$$

$$A_y = -2.67\text{ kN}$$

$$\Sigma F_z = 0: \quad A_z + \frac{6}{11}(5.24\text{ kN}) - \frac{6}{11}(5.24\text{ kN}) = 0$$

$$A_z = 0 \qquad \mathbf{A} = (5.72\text{ kN})\mathbf{i} - (2.67\text{ kN})\mathbf{j} \blacktriangleleft$$

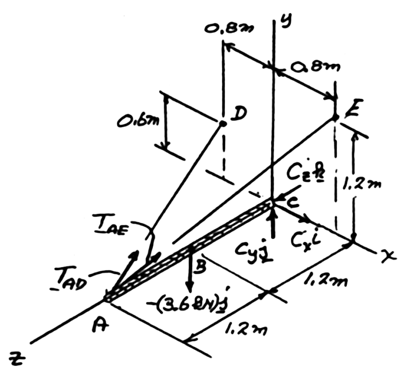


PROBLEM 4.105

A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).



$$\mathbf{r}_B = 1.2\mathbf{k}$$
$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$
$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$
$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0\text{:} \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}\text{:} \quad -0.55385T_{AD} - 1.02857T_{AE} + 4.32 = 0 \qquad (1)$$
$$\mathbf{j}\text{:} \quad -0.73846T_{AD} + 0.68671T_{AE} = 0$$

$$T_{AD} = 0.92857T_{AE} \qquad (2)$$

From Eq. (1):

$$-0.55385(0.92857)T_{AE} - 1.02857T_{AE} + 4.32 = 0$$

$$1.54286T_{AE} = 4.32$$
$$T_{AE} = 2.800 \text{ kN} \qquad T_{AE} = 2.80 \text{ kN} \blacktriangleleft$$

PROBLEM 4.105 (Continued)

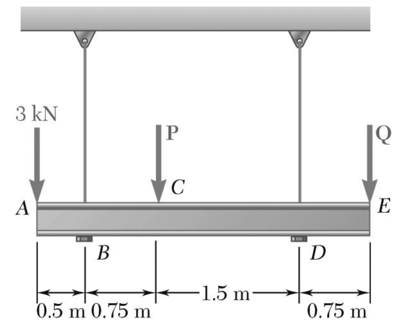
From Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600 \text{ kN}$ $\qquad T_{AD} = 2.60 \text{ kN}$ ◀

$$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(2.6 \text{ kN}) + \frac{0.8}{2.8}(2.8 \text{ kN}) = 0 \qquad C_x = 0$$

$$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(2.6 \text{ kN}) + \frac{1.2}{2.8}(2.8 \text{ kN}) - (3.6 \text{ kN}) = 0 \qquad C_y = 1.800 \text{ kN}$$

$$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(2.6 \text{ kN}) - \frac{2.4}{2.8}(2.8 \text{ kN}) = 0 \qquad C_z = 4.80 \text{ kN}$$

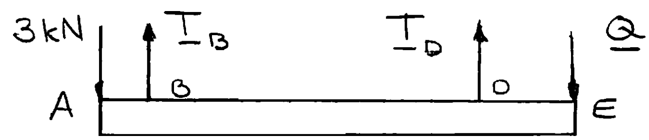
$\mathbf{C} = (1.800 \text{ kN})\mathbf{j} + (4.80 \text{ kN})\mathbf{k}$ ◀



PROBLEM 4.11

Three loads are applied as shown to a light beam supported by cables attached at B and D. Neglecting the weight of the beam, determine the range of values of Q for which neither cable becomes slack when $P = 0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (3.00 \text{ kN})(0.500 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.500 \text{ kN} + (0.750)\, T_D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (3.00 \text{ kN})(2.75 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 11.00 \text{ kN} - (3.00)\, T_B \qquad (2)$$

For cable B not to be slack, $T_B \geq 0$, and from Eq. (2),

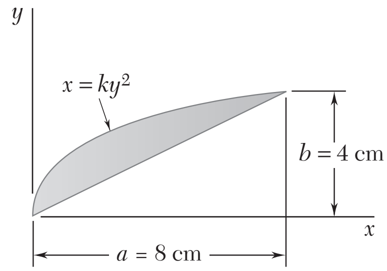
$$Q \leq 11.00 \text{ kN}$$

For cable D not to be slack, $T_D \geq 0$, and from Eq. (1),

$$Q \geq 0.500 \text{ kN}$$

For neither cable to be slack,

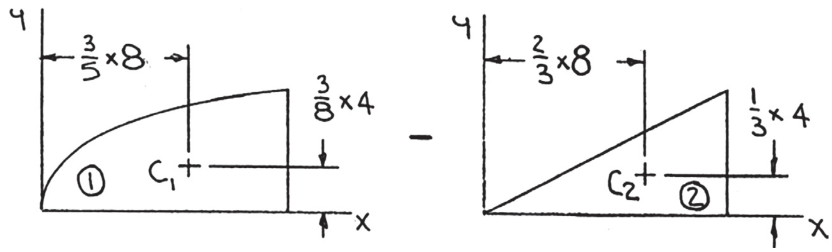
$$0.500 \text{ kN} \leq Q \leq 11.00 \text{ kN} \blacktriangleleft$$



PROBLEM 5.14

Locate the centroid of the plane area shown.

SOLUTION



Dimensions in cm

	A, cm^2	$\bar{x}$, cm	$\bar{y}$, cm	$\bar{x}A$, cm^3	$\bar{y}A$, cm^3
1	$\frac{2}{3}(4)(8) = 21.333$	4.8	1.5	102.398	32.000
2	$-\frac{1}{2}(4)(8) = -16.0000$	5.3333	1.33333	−85.333	−21.333
Σ	5.3333			17.0650	10.6670

Then $$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(5.3333\text{ cm}^2) = 17.0650\text{ cm}^3 \qquad \text{or} \quad \bar{X} = 3.20\text{ cm} \blacktriangleleft$$

and $$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}(5.3333\text{ cm}^2) = 10.6670\text{ cm}^3 \qquad \text{or} \quad \bar{Y} = 2.00\text{ cm} \blacktriangleleft$$